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The Society have received from Mr. F. Abbott a communication, entitled "Some further Observations on the Variable Star η *Argûs* and the Surrounding Nebula," dated Hobart Town, Tasmania, 15th February, 1870, accompanied by a drawing of the nebula taken 28th January, 1870. He refers to his former drawings, 1863, *Monthly Notices*, vol. xxiv. p. v., and 1868, ditto, vol. xxviii. p. 200, and to the *Cape Monograph*, and remarks that the alterations which have taken place in the nebula since 1868 will be at once seen on an inspection of the drawings and by comparing them with each other. In the *Cape Monograph* the dark space is an inclosure. In 1863 it had two openings, one at each end. In 1868 there were four openings, and now in 1870 there are five, exposing a number of isolated and distinct stars, which render it difficult but from position to know which is the star η *Argûs*. There is attendant on these changes an increase of light which is notable up to the present time.

On a Property of the Stereographic Projection.

By Prof. Cayley.

I am not aware whether it has been noticed that the very same circles which in the direct stereographic projection of a hemisphere (viz., that wherein the projection is on the plane of a meridian) represent the meridians and parallels respectively, —represent also in the oblique projection of the hemisphere meridians and parallels respectively. In fact, in the direct projection where the poles N, S, are in the horizon-meridian, or bounding circle of the projection, if we take a chord A B at right angles to N S, and on A B as diameter describe a circle, the original (meridian and parallel) circles will, as the appearance of the figure at once suggests, represent meridians and parallels in the oblique projection in which the horizon or bounding circle of the projection is the circle diameter A B, and where consequently the North Pole N is brought into view, the South Pole S being beyond the limits of the projection. That this really is so, is clear from the consideration that in any stereographic projection whatever, the meridians will be circles passing through two fixed points N, S, and the parallels be circles cutting the meridians at right angles. (Or, what is the same thing, the parallels also pass each of them through two fixed imaginary points, the antipoints of N, S, but this in passing.) And, moreover, since in the oblique, as well as in the direct projection, the longitude of any meridian, as reckoned from the central meridian N S, is the angle at N between the two meridians, the longitude for a given meridian is the same in the two projections respectively. But the co-latitudes are not the same in the two projections respectively; viz., a circle which in the

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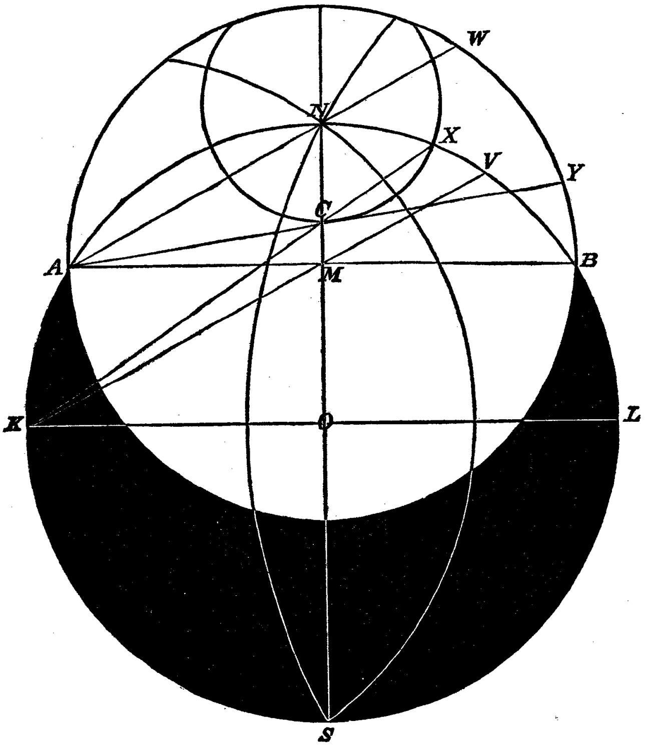
direct projection represents the parallel co-latitude c, will in the oblique projection represent the parallel of a different co-latitude c'. The relation between the values of c, c', will of course depend upon the position of the bounding circle A B of the oblique direction: to define this position, we may use either the arc N M which in the direct projection determines the co-latitude of the centre M of the oblique projection (say N M $= \Delta$, that is, N V $= \Delta$), or by the arc N M which in the oblique projection determines the distance of N from the centre, or co-latitude of the centre (say N M $= \Delta'$, that is, B W $= \Delta'$.) The obliquity in the oblique projection is thus $90^\circ - \Delta'$, viz., this is the inclination of the plane of projection to that of the horizon-meridian in the direct projection. We have also $c =$ N X, $c' =$ W Y. The relation between the angles Δ, Δ', is easily found to be



$$\tan \tfrac{1}{2}\Delta = \tan^2 \tfrac{1}{2}\Delta',$$

viz., taking the radius in the direct projection to be $= 1$, we have

$$\text{O M} = \tan \tfrac{1}{2}(90^\circ - \Delta),$$

$$\text{M A} = \sqrt{1 - \tan^2 \tfrac{1}{2}(90^\circ - \Delta)},$$

$$\text{M N} = 1 - \tan \tfrac{1}{2}(90^\circ - \Delta);$$

wherefore

$$\sqrt{1 - \tan^2 \tfrac{1}{2}(90^\circ - \Delta)} \;.\; \tan \tfrac{1}{2}\Delta' = 1 - \tan \tfrac{1}{2}(90^\circ - \Delta).$$

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and thence

$$\tan^2 \tfrac{1}{2}\Delta' = \frac{1 - \tan\frac{1}{2}(90^\circ - \Delta)}{1 + \tan\frac{1}{2}(90^\circ - \Delta)} = \tan\tfrac{1}{2}\Delta,$$

the required relation.

We have moreover

$$\begin{aligned} NC = 1 - \tan\tfrac{1}{2}(90^\circ - c) &= AM\left\{\tan\tfrac{1}{2}\Delta' - \tan\tfrac{1}{2}(\Delta' - c')\right\} \\ &= \sin\Delta'\left\{\tan\tfrac{1}{2}\Delta' - \tan\tfrac{1}{2}(\Delta' - c')\right\} \\ &= 2\sin^2\tfrac{1}{2}\Delta' - \sin\Delta' \tan\tfrac{1}{2}(\Delta' - c'), \end{aligned}$$

that is

$$\begin{aligned} \tan\tfrac{1}{2}(90^\circ - c) &= \cos\Delta' + \sin\Delta' \tan\tfrac{1}{2}(\Delta' - c') \\ &= \frac{\cos\frac{1}{2}(\Delta' + c')}{\cos\frac{1}{2}(\Delta' - c')}, \end{aligned}$$

or, what is the same thing,

$$\frac{1 - \tan\frac{1}{2}c}{1 + \tan\frac{1}{2}c} = \frac{1 + \tan\frac{1}{2}c' \tan\frac{1}{2}\Delta'}{1 + \tan\frac{1}{2}c' \tan\frac{1}{2}\Delta'},$$

that is

$$\tan\tfrac{1}{2}c = \tan\tfrac{1}{2}\Delta' \tan\tfrac{1}{2}c',$$

which is the required relation between c and c'. In the particular case $\Delta = \Delta' = 90^\circ$, the two projections coincide, and we have, as we should do, $c' = c$.

Note respecting Solar Spots visible to the Naked Eye.

By A. R. Hill, Sub. Lieut. R.N.

On Sunday, 22^{d} May, at from about 5 to 6.30 P.M., the extreme light of the Sun being obscured by a peculiar scud drifting over it, and giving the whole disk a reddish appearance, with the borders less luminous than the centre, I observed three large spots, A, B, and C, as in diagram, fig. 1, distinctly visible with the naked eye, especially A and B, A being the most distinct, and C being only visible at times, when the ardour of the Sun's rays was diminished more than at others.

On Monday, 23d, at about the same time P.M., the atmosphere being still in the same state as on the previous day, but more advantageous for naked-eye observations, I observed the spots, A, B, and C again, C being more distinct than on the previous evening; and also another spot, D, fig. 2.; but this latter being only visible at the most advantageous intervals.